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Observations on the Thermally Induced Twist of Thin-Walled Open-Section Booms

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Nomenclature

$BM_x(z)$ = $e_cEh\int_0^p \tilde{T}(s,z)\cdot Y_c(s)\cdot ds$, the thermal bending moment about X axis
$BM_y(z)$ = $e_cEh\int_0^p \tilde{T}(s,z)\cdot X_c(s)\cdot ds$ the thermal bending moment about Y axis
C = torsional rigidity of boom
C_1 = warping rigidity of boom
E = Young's modulus
e_c = coefficient of thermal expansion
$\mathbf{e}_{sc}$ = vector from the shear center to the surface element ds
$f_0(z),f_1(z),f_2(z)$ = functions of z
I_x = $\int_0^p hY_c^2(s)\cdot ds$
I_y = $\int_0^p hX_c^2(s)\cdot ds$
$\tilde{T}(s,z)$ = local temperature
$\tilde{T}_A$ = ambient temperature
$\tilde{T}_m(z)$ = $\frac{1}{p}\int_0^p \tilde{T}(s,z)\cdot ds$, the section averaged temperature
$T_{sc}(z)$ = thermally induced torque
$\epsilon_z(s,z)$ = thermal strain in z direction
$\phi(L)$ = angular response at tip
$\sigma_z(s,z)$ = normal stress in z direction
$\tau(s,z)$ = shear stress in sz plane

Introduction

STRUCTURAL considerations during launch and attitude maneuvers of spacecraft necessitate the use of mechanical subsystems that can be deployed and retracted in orbit. Open sections booms (STEMs)[1] are utilized as communication antennas, gravity gradient stabilizing booms, or as structural components in package transfer devices, solar panel actuators, and instrument carriers. In some applications, it is necessary that the open section boom have minimum twisting, and the various sources including solar heating, that induce twisting have to be identified and evaluated. A particular requirement of this knowledge is for the mass spectrometer experiments intended on Apollo 15 and 16 lunar missions. The mass spectrometer is mounted at the tip of a 25-ft-long BI-STEM† boom and requires a nominal pointing accuracy of ±15°.

Thermal distortion (bending and twisting) occurs in booms due to the interactions of both temperature distributions and mechanical end conditions. Thermal effects cause the boom to distort to its minimum strain energy position. Extensive work has been carried out by many investigators in predicting both the static and dynamic behavior of thin-walled cylinders of open sections when exposed to solar radiation.

The prerequisite to predicting the steady-state behavior of thin-walled open sections is to model the complex heat-transfer characteristics at the interfaces. This hypothetically, enables the thermoelastic equations to be solved simultaneously with the thermal equilibrium equation. However, the latter equation is nonlinear and further simplifying assumptions have to be made. The work of Florio and Hobbs[2] in solving the thermal equilibrium equation is significant. Subsequently, the work of Graham[3] has been used in solving the more general case which includes heat transfer by internal radiation and interface conduction in overlapped open section booms. The numerical solution of this equation will yield results that are correct to the extent that the assumed modelling is justified. This ambiguity in the results may necessitate experimental investigation to be carried out on the boom. The variations in the lengths of flight booms range from a few meters to 200. In addition, the gravitational problems in simulating the environment of outer space among a host of others restrict the size of the test apparatus.

Experimental investigations thus pose the problem of scaling the results to the desired length of the boom. The objective of this Note is to re-examine the problem of thermally induced twist of open section booms with zero isothermal twist, and present rapid methods of predicting their steady state behavior. Analytical, physical and experimental considerations are utilized for arriving at the values of thermal twist.

Thermally Induced Torque

Restricting the discussion to a circular thin-walled open section boom (Fig. 1), the thermally induced torque $T_{sc}(z)$ may be expressed as

$$\mathbf{T}_{sc}(z) = -\int_0^p \mathbf{e}_{sc} \times h\boldsymbol{\tau}(s,z)\cdot ds \quad (1)$$

For equilibrium in the thin-walled member the shear stress $\tau(s,z)$ is related to the longitudinal stress $\sigma_z(s,z)$ by the relationship

$$\partial h\cdot\tau(s,z)/\partial s = -h\partial\sigma_z(s,z)/\partial z \quad (2)$$

The thermo-elastic equation[4] for the stress $\sigma_z(s,z)$ is

$$\sigma_z(s,z) = E\{\epsilon_z(s,z) - e_c[\tilde{T}(s,z) - \tilde{T}_A]\}$$

With the implicit assumption that plane sections remain plane by the expression for $\epsilon_z(s,z)$ that

$$\epsilon_z(s,z) = f_0(z) + f_1(z)X_c(s) + f_2(z)Y_c(s)$$

the relationship for $\sigma_z(s,z)$ may be obtained[5] to satisfy the equilibrium conditions about the principal axes X and Y as

$$\sigma_z(s,z) = E\cdot e_c[\tilde{T}_m(z) - \tilde{T}(s,z)] - [BM_y(z)/I_y]\cdot X_c(s) + [BM_x(z)/I_x]\cdot Y_c(s) \quad (3)$$

Partially differentiating equation, Eq. (3), with respect to z, we see that for a particular s, the longitudinal distribution of the temperature $\tilde{T}(s,z)$ controls the value of $\partial\sigma_z(s,z)/\partial z$ as $BM_x(z)$ and $BM_y(z)$ are also functions of $\tilde{T}(s,z)$. For a uniform open section boom, whose orientation is constant along

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† BI-STEM: Proprietary name for two slit thin walled cylinders nested with seams in opposition.

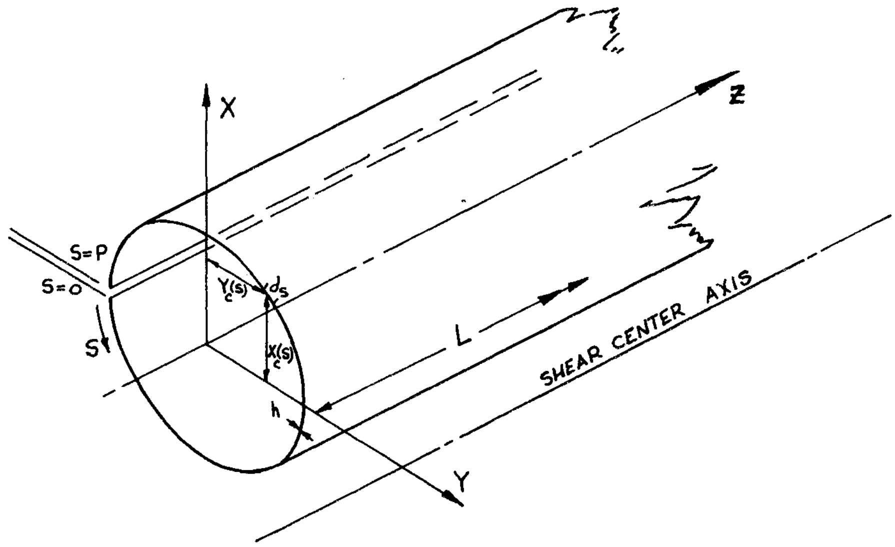

Fig. 1 Geometrical nomenclature of open-section boom.

its length with respect to the sun, it would be expected that the longitudinal temperature distribution will be predominantly constant along rays of constant s. This would then imply from Eq. (3), that

$$\partial\sigma_z(s,z)/\partial z = 0 \tag{4}$$

Referring to Eq. (2), Eq. (4) would imply that $\tau(s,z)$ = a constant around the periphery, and to satisfy the free edge boundary condition, it would have to be uniformly zero. Hence the thermally induced twist would be zero. Predmore[6] has carried out thermal deflection tests on open section booms in a side-on sun simulated vacuum test chamber, which can accommodate boom lengths up to three meters. However, experimental findings of Predmore are contrary to these predictions. This anomaly may be explained by clarifying the work of Jordan,[7] whose work implies that Eq. (3) is valid only if plane sections remained plane after thermal distortion. This implies that mechanical end conditions also control the thermally induced twisting of open section booms.

Fixed/free boom

The implications of Jordan's analysis is that although the temperatures may be constant along rays of constant s on a zero pretwist open section, thermally induced twisting invariably occurs in a boom that is fixed at one end and free at the other. His arguments can be understood when it is realized that the peripheral temperature distributions may be such in an open section, that at final equilibrium, plane sections may not remain plane. This is due to the discontinuity in the peripheral temperature distribution at the free edges, which makes the open section to warp (see Fig. 2). For a fixed/free boom, the open section at the fixed end is constrained to remain plane. This is equivalent to a normal stress distribution $\sigma_z(s,z)$ being applied to give zero warp at the fixed end. Along constant s rays of the boom, $\sigma_z(s,z)$ reduces to zero in some manner at the free end. Assuming that this reduction of $\sigma_z(s,z)$ occurs linearly, the shear stress $\tau(s,z)$ distribution will be similar around the periphery [see Eq. (2)] at each longitudinal location. Because the shear stress is zero along the free edges, the magnitude of the shear stress will be equal at each longitudinal location along constant s. Its peak value is controlled by the distribution of $\partial\sigma_z(s,z)/\partial z$ around the periphery. If this distribution gives rise to a resultant shear force whose line of action does not pass through the shear center, thermally induced twisting will occur as though a constant mechanical torque has been applied along the length. The final response obviously will not be as explained, because of temperature pattern changes as soon as the boom ceases to have zero twist.

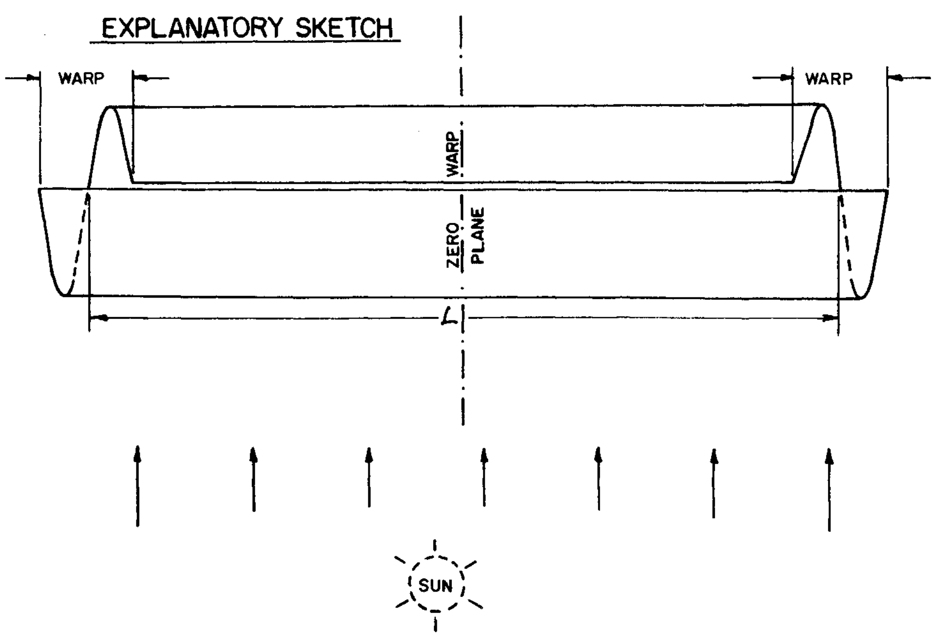

Fig. 2 Thermally induced warp (neglecting thermal bending and twisting).

The magnitude of the resultant shear force for the open section depends strongly on the peripherally weighted gradient $\partial\sigma_z(s,z)/\partial z$. An increase in the length of the boom leads to a decrease in this gradient because the $\sigma_z(s,z)$ distribution remains the same at the fixed end independent of the length. This implies that the magnitude of the thermal torque $T_{sc}(z)$ depends inversely on the length of the open section boom for each sun orientation.

By mechanical analogy, for a fixed/free open section boom: 1) for $CL^2/C_1 \ll 1$, the tip response $\phi(L) \simeq T_{sc}(z)\cdot L^3/3C_1$, hence $\phi(L) \simeq K_1L^2$, where K_1 is nearly a constant; 2) for $CL^2/C_1 \gg 1$, let $a = (CL^2/C_1)^{0.5}$. The mechanical torsion equation[8] for an open section boom is

$$T_{sc}(z) = C(d\phi/dz) - C_1(d^3\phi/dz^3)$$

The solution for a fixed/free boom can be obtained as

$$\phi(z) = \frac{T_{sc}(z)\cdot L}{C}\left\{\frac{Z}{L} - \frac{1}{a}\left[\sinh\frac{az}{L} - (\tanh a) \times \left(\cosh\frac{az}{L} - 1\right)\right]\right\}$$

for $CL^2/C_1 \gg 1$, implies $a \gg 1$ then $\phi(L) = (T_{sc}(z)\cdot L/C)$ $[1 - \text{Order } (1/a)]$ $\therefore$ the tip response $\phi(L) \rightarrow A$ constant, as $T_{sc}(z)$ is proportional to $1/L$.

Fixed/fixed boom

In an open section boom that is fixed at either end (i.e., prevented from warping), the stress distribution $\sigma_z(s,z)$ will be similar at both ends.

This implies that

$$\partial\sigma_z(s,z)/\partial z = 0 = \partial\tau(s,z)/\partial s$$

It would be expected that the shear stress, to satisfy the free edge boundary conditions will be uniformly zero at all points in the boom. Hence no twist should occur. This conclusion has also been reached by Jordan.[7]

Free/free boom

The thermal twist of an open section boom that is free to warp at either end may be regarded as two fixed/free booms, fixed symmetrically at the central plane. Relative to the central plane, the two halves of the boom twist in the same direction and manner as fixed/free open section booms, and the earlier discussion for a fixed/free boom becomes applicable. Boom sections equidistant from the central plane have zero angular displacement relative to each other.

Concluding Comments

The foregoing discussion used a circular open-section boom because of its conceptual simplicity for the illustration of thermally induced twist. The arguments and conclusions presented are equally applicable for other open-section booms. Actual flight booms (STEMs and BI-STEMs) have a certain amount of overlap whose physical properties of heat flow paths and interface friction may vary unpredictably along the length. However, because of their random nature, the assumption of their uniformity is justified as the boom length increases. Also, the interface friction introduces a resisting torque that the thermal torque $T_{sc}(z)$ must exceed before the boom behaves like an open-section one. The implication of this observation is that for fixed/free or free/free overlapped open-section booms greater than certain lengths, the thermal torque is negligibly small. Under the influence of a thermal torque that exceeds the friction torque, an open-section boom's response depends ideally on its end conditions. It is stressed that the suggested method of extrapolation from experimental results is accurate only for small angles of thermally induced twist, nominally, less than 45°.

Finally, on the basis of this article, a comment on the excellent paper by Frisch[9] is in order. The boundary condition $\partial^2\phi/\partial z^2|_{z=L} = 0$ (Eq. 54 of Ref. 9), is valid for the analysis, contrary to Jordan's[7] postulation for a fixed/free boom. The anomaly with the experimental evidence and Jordan's work lies in Eq. (1),[9] which has been applied with the implicit assumption that plane sections remain plane after the open-section thermally distorts. To satisfy the equilibrium conditions with which the expression for $\sigma_z(s,z)$ has been derived,[5] the boom will have already twisted. In this position, $\partial^2\phi/\partial z^2|_{z=L} = 0$ at the free end.

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Radiosity at the Midpoint of Parallel Plates

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Nomenclature

$B(X)$	=	nondimensional radiosity
h	=	ratio of the plate separation distance to the plate length
H	=	parameter associated with the plate separation distance
$K(X,Y)$	=	kernel of the Fredholm equation
L	=	length of the parallel plates
$R(X)$	=	radiosity
T	=	temperature of the parallel plates
u	=	dummy variable of integration
γ	=	parallel plate separation distance
ϵ	=	emittance
ρ	=	reflectance
σ	=	Stefan-Boltzmann constant

Introduction

THE Fredholm equation of the second kind that governs the radiative transfer between a parallel plate configuration does not lend itself to analytical treatment as the separation distance approaches zero. Approximate numerical approaches do not converge rapidly for this condition, especially when the emittance, ϵ, is small. This note is con-

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